Do internal capital markets in business groups mitigate firms financial constraints?

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Abstract

We develop a new rationale for capital allocation in business groups' internal capital markets. We show that productivity and pledgeable income jointly drive capital allocation within an internal capital market. In financially constrained business groups, an efficient internal capital market can allocate marginal funds to firms that have high pledgeability because of a multiplier effect: a dollar of internal funds generates a bigger increase in investment. This result has important implications for the business group affiliation strategy. Whether or not a financially constrained but highly productive firm will benefit from group affiliation depends on its borrowing capacity vis-à-vis other affiliates.

1 Introduction

Business groups—groups of legally independent private and publicly listed firms with limited liability and autonomous access to external capital markets—are present around the world, including emerging economies in Latin America (Chong and López-de Silanes, 2007) and Asia (Claessens et al., 2002; Carney and Child, 2012) and developed economies in Europe (Faccio and Lang, 2002). In a business group, a controlling shareholder—an individual, a founding family, or the state—controls firms through a pyramidal organizational structure, that is, a chain of ownership relations in which the controlling shareholder directly controls a firm that, in turn, controls another firm, and so on (La Porta et al., 1999; Almeida and Wolfenzon, 2006a). This structure allows the controlling shareholder at the top of the pyramid to achieve legal control of the decisions in the firms down the ownership chain owning only a small amount of cash flow rights (Berle and Means, 1932), fundamentally differing from the conglomerates or multidivisional organizations of fully owned subsidiaries or divisions.

In spite of the fundamental differences between business groups and conglomerates, and following Stein's (1997) influential work, most theoretical models of internal capital markets have focused on conglomerates (with a few exceptions, e.g., Cestone and Fumagalli (2005)), which are commonplace in the United States (Kandel et al., 2018), but not elsewhere (La Porta et al., 1999). We also know that the role of the laws aimed at protecting outside investors from looting of firms by controlling shareholders and the ways the courts apply them have important implications for firms' organization forms (Johnson et al., 2000; La Porta et al., 1999; Belenzon et al., 2018).

This article attempts to model the allocation of internal resources in business groups under different investor protections scenarios, aiming to shed new light on the effects of the idiosyncrasies of the business groups' organizational structure on corporate finance. More specifically, we aim to answer the following questions: (a) How do business groups allocate resources in their internal capital markets? (b) Do the internal capital markets alleviate the

financial constraints of affiliate firms that have limited access to external finance?, and (c) What is the role of investor protection for internal capital markets in business groups?

The independence of business groups' affiliated firms allows them to directly access external capital markets and to secure financing on their own merits. We contend that the resource allocation within a business group could be related to the same factors that drive resource allocation in external capital markets. We therefore develop a simple model of investment in business groups subject to moral hazard, proposing that a firm's productivity and pledgeable income (external financing capacity) jointly explain the (efficient) allocation of internal resources in business groups. The central result is that, if two companies have different amounts of pledgeable income, it could be better to allocate resources to the firm with the greatest ability to multiply its wealth than to the most productive firm.

Thus, financially constrained business groups could decide to finance investment opportunities that allow them to increase their external financing—that is, where internal financing has a multiplier effect over external finance—instead of financing the most profitable opportunities. In other words, both winner picking and cross-subsidization strategies for internal capital markets discussed in the corporate finance literature can arise, but for a different reason.

For example, suppose that firms 1 and 2 belong to a business group. Firm 1 has an investment with a net present value (NPV) of \$0.15 per unit of a dollar and can raise \$0.80 from outside investors per unit of internal wealth. Firm 2 has an investment with a NPV of \$0.20 per unit of a dollar and can raise \$0.30 from outside investors per unit of internal wealth. If an entrepreneur has equal cash flow rights in both firms, the entrepreneur maximizes wealth by allocating the maximum possible amount of internal resources to firm 1. For each \$1 of internal wealth, firm 1 generates an economic surplus of $\$0.27 = (1 + 0.8) \times 0.15$, whereas firm 2 generates a surplus of $\$0.26 = (1 + 0.3) \times 0.20$.

This example illustrates our central result in two ways. First, productivity alone should not explain the resource allocation of internal capital markets within a business group. Sec-

ond, pledgeable income is an important factor (if not the most important) in financing investments across firms within business groups.

In other words, our model's predictions question the argument that the efficiency of an internal capital market is related to the allocation of resources to high-productivity firms alone. Our key point is that, in a financially constrained business group, an efficient internal capital market can allocate marginal funds to firms that have high pledgeability because of a credit multiplier effect, where a dollar of internal funds generates a bigger increase in investment. In other words, we propose that an efficient internal capital market in constrained business groups can allocate resources from financially weak to financially strong firms, or, more precisely, from firms with low multipliers to firms with high multipliers.

According to our model, if productivity varies little relative to pledgeable income across firms in the same business group, pledgeable income tends to be the most critical driver of resource allocation within business groups. Thus, if one analyzes the determinants of the resource allocation, taking pledgeable income for granted, one could conclude that the internal capital markets are inefficient. This conclusion is especially troubling if there is a negative correlation between productivity and pledgeable income (as in the example above).

Indeed, Shin and Park (1999) and Lee et al. (2009) have concluded that internal capital markets do not improve the efficiency of resource allocation in Korean business groups (*chaebols*), showing that chaebols invest more than their non-chaebol counterparts in firms with poor growth opportunities (i.e., low-productivity firms). However, this research does not exclude the alternative explanation of the multiplier effect of internal financing over external funds, as the authors do not control for variables associated with pledgeable income, such as private benefits of control, tangible assets, and risk shifting. In other words, the authors conclude that the internal capital markets of chaebols are inefficient, whereas, in reality, the reduction of financial constraints for the group as a whole could result in an efficient outcome.

Our contribution to the corporate finance literature is twofold. First, we build a new

model showing that internal capital markets in business groups resemble external financial markets. In other words, our model implies that the same factors that limit a firm's access to external finance also reduce its access to financial resources in internal capital markets. According to the literature, a company that has considerable private benefits, few tangible assets (i.e., collateral), and/or high risk-shifting problems can have difficulties raising external finance (e.g., (Stiglitz and Weis, 1981; Bernanke and Gertler, 1989; Gertler and Gilchrist, 1994; Holmstrom and Tirole, 1997; Kiyotaki and Moore, 1997; Almeida and Campello, 2007)). We propose that, if a firm with the same above-mentioned characteristics is affiliated with a business group, it will also face financial constraints in the internal capital market. More specifically, this focal firm is likely to be a provider (and not a receiver) of finance for (from) other companies in the business group. This theoretical prediction contradicts the cross-subsidization view in which the internal capital market of business groups can mitigate the negative effect of the failure of external financial markets (Khanna and Palepu, 2000; Khanna and Yafeh, 2007).

Second, our model generates new testable predictions. For example, we distinguish between receivers and providers of intra-group loans. Only the investment of receivers is sensitive to other affiliates' cash flow, because receivers benefit from the internal capital market whereas providers support it. As pledgeable income enables firms to multiply internal wealth and increase investment spending, the investment sensitivity to other affiliates' cash flow tends to be positive and to increase with pledgeable income.

Moreover, the likelihood that a firm will receive intra-group loans increases with productivity, pledgeable income, and controlling shareholder cash flow rights. In other words, the same factors that make a firm a good candidate for external finance also increase its odds of accessing the internal capital market. Finally, if financially strong firms are those that receive resources from other affiliates in business groups, these firms will be able to invest more than their standalone counterparts. On the flip side, if the financially weak firms in a business group tend to support the internal capital market by sharing their positive cash

flow with other affiliates, these firms will have fewer resources available, and, consequently, they will invest less than similar non–business group firms.

Our main theoretical prediction also finds support in the empirical literature. For example, in Chilean business groups, Buchuk et al. (2014) show that net receivers of intra-group loans tend to be the firms with the most growth opportunities (Tobin's Q), high asset tangibility (property, plant, and equipment), and small size. This evidence is consistent with the prediction that productivity (growth opportunities) and pledgeable income (asset tangibility) jointly determine the allocation of internal resources in business groups.[1]

The article proceeds as follows. In the next section, we discuss the related literature on resource allocation in business groups' internal capital. Next, in Section 3, we develop our model for financial resource allocation in business groups. In Section 4, we discuss the key results and propose testable hypotheses. Finally, Section 5 concludes the article.

2 Related literature

Our article rests on both theoretical and empirical research on resource allocation in internal capital markets. From a theoretical standpoint, Stein (1997) develops an investment model in conglomerates (multidivisional firms) in which headquarters with proper incentives and power to freely transfer resources between divisions engage in a winner-picking strategy, taking scarce funds from low-productivity divisions to give to high-productivity divisions, improving overall performance. Although, a winner-picking strategy could emerge in our model, this will not always be the case. In our model, productivity and pledgeable income interact to determine internal resource allocation, and, in some cases, it will be optimal for business groups to allocate more resources to an affiliated firm with higher pledgeable income, even if this unit is not the most productive. These differences in predictions arise

[1] Gopalan et al. (2007) show that net intra-group loans are insensitive to growth opportunities and decrease with the degree of asset tangibility in Indian business groups. Differently, Chilean firms operate in an institutional environment whose structure resembles our model's, which partly explains the conflicting findings in the empirical literature on internal capital markets.

because business group affiliates are independent legal entities and must approach outside investors based on their own merits, whereas, in Stein's (1997) model, the conglomerate's headquarters approaches outside investors for funding and then allocates resources across divisions, with the entire conglomerate being liable for the debt repayment.

In addition, Cestone and Fumagalli (2005) provide one of the first attempts to model the resource allocation decisions in business groups' internal capital markets.[2] Our model shares several of their assumptions[3] and, in a broader sense, some of the results. For example, in both our model and theirs, winner picking, and cross-subsidization can arise in business groups' internal capital markets. However, there are also remarkable differences. In Cestone and Fumagalli's model, the outcome depends mainly on the amount of internal wealth available (the intensity of financial constraints at the group level), winner picking (cross-subsidization) being more likely if the business group suffers strict (loose) financial constraints. In our model, the direction of resources in the internal capital market depends mainly on each group's affiliated firms' characteristics, such as private benefits of control, asset tangibility, risk-shifting problems, controlling shareholder cash flow rights, and productivity.

The differences in allocations in business groups between Cestone and Fumagalli's (2005) and our model arise because of different assumptions regarding the investment decisions of the business group affiliates. First, Cestone and Fumagalli (2005) assume a fixed investment size, whereas we opt for a model with continuous investment. This explains why their results are based on the amount of internal wealth and ours is not (as long as there are financial constraints). Second, in Cestone and Fumagalli's model, as moral hazard is in the form of costly and unobservable managerial effort, productivity and pledgeable income go hand in hand; that is, the most productive firm is also the one with the highest income to pledge to

[2]The main focus of this study is the interaction between internal capital markets and product market competition.

[3]For example, after receiving their internal capital allocations, business groups members raise additional resources from outside investors for investment; most importantly, the rest of the group is not liable for this external debt.

outside investors. In our model, we disentangle these constructs, allowing pledgeable income to interact with other variables besides productivity. This approach explains why, in our case, the direction of resources in the internal capital market depends on all these group-affiliated firm variables and why resources can be shifted to firms of low productivity but high pledgeable income. In this sense, our model is innovative because it shows how productivity and variables related to financial capacity interact to determine resource allocation in business groups' internal capital markets, extending prior literature on conglomerates Stein (1997) and business groups Cestone and Fumagalli (2005).

Lastly, Samphantharak (2006) develops a dynamic investment model for business groups with costly external finance. In that model, assuming that a controlling shareholder can freely transfer resources within the group, including funds raised in the external financial markets, all firms in the group will borrow until their marginal costs of external finance are equal, giving rise to an "insurance effect" across affiliated firms. That is, through internal transfers, the entire business group absorbs an idiosyncratic shock affecting the cost of external finance in one particular firm. These transfers also give rise to a "tunneling effect" in which firms with lower costs of external finance provide resources to firms with higher costs of capital. These predictions confirm prior literature on business groups that claims that internal capital markets can mitigate firms' financial constraints (Khanna and Palepu, 2000; Khanna and Yafeh, 2007).

Our model diverges from that of Samphantharak (2006) in several ways. First, our assumptions make external finance (and investment) proportional to the internal resources available to each group firm, whereas Samphantharak (2006) uses a costly external borrowing approach in which external finance becomes more expensive as it increases, but its availability is not directly related to the amount of internal resources available to each affiliated firm. In short, our financial constraints are in terms of *quantity* and those of Samphantharak (2006) are in terms of *cost*.

Second, neither the insurance effect nor the tunneling effect is included in our model. In

contrast, in our model, if a firm is hit by an external shock that affects its level of pledgeable income, the shock reduces the firm's likelihood of obtaining resources in the internal capital market. In other words, the external shock is amplified within the business group. Third, in terms of the assumptions, there are also differences between our model and that developed by Samphantharak (2006). In our model, we do not allow joint responsibility for loans. Each group firm has its own budget constraint and must raise external finance based on its own merits. We also require that resource allocation within the business group be carried as group firm liability for repayment. Specifically, if a firm receives resources today, it must repay an (interest-adjusted) amount in the future. In Samphantharak's (2006) model, the affiliated firm has no liability to repay, and the controlling shareholder can freely shift resources across group firms, as long as they add up to zero. This assumption makes the business group behave as if it had only one joint business constraint.[4] Altogether, these differences lead to different outcomes and implications for the resource allocation and efficiency in business groups' internal capital markets, as discussed in the next section.

From an empirical perspective, this study relates to those of Almeida and Wolfenzon (2006b,a). Assuming that internal capital markets mitigate the limited pledgeability problem that characterizes external financial markets, Almeida and Wolfenzon (2006b) show that conglomerates' internal capital markets can reduce the efficiency of economy-wide capital allocation. This result is especially salient in countries with intermediate levels of investor protection. Even though we do not look for such an economy-wide equilibrium effect, our model suggests that internal capital markets in business groups could bear the same characteristics as external markets. That is, internal capital markets in business groups might not mitigate the limited pledgeability problem. If this is the case (as we predict), there could be an even greater loss of efficiency in economy-wide capital allocation than noted by Almeida and Wolfenzon (2006b).

Finally, Almeida and Wolfenzon (2006a) provide a theoretical rationale for the formation

[4]It is worth noting that the power to freely shift resources in the internal capital markets can harm the interests of outside investors of the donor firm.

of pyramidal ownership in family business groups. They show, for example, that family business groups should be more common in countries with low levels of investor protection, because families can use resources from firms they already control to finance new ones. The authors argue that this financing advantage over other entrepreneurs is more important in countries with weak investor protection, where pledgeable income tends to be lower. Almeida and Wolfenzon (2006a) suggest that financial factors can foster the formation of family business groups in weak–investor protection environments. Our model suggests that these same factors could be the key drivers of resource allocation in the internal capital markets in business groups.

3 The Model

We develop a simple model in the spirit of Tirole (2006) to derive empirical implications about the investment behavior and external/internal financing in business group–affiliated firms. We propose a one-period model in which a risk-neutral entrepreneur entirely (and directly) owns a firm U (up). Firm U, along with outside investors (also risk neutral), owns a second firm, called D (down). An entrepreneur controls these two firms and owns a fraction β of the capital (economic rights) of firm D (directly and indirectly through firm U).

The entrepreneur is assumed to retain control over firm D, whatever the size of β.[5] On date 0, both firms have opportunities to invest. If firm U invests I^U on date 0, it will receive a cash flow of $K^U I^U$ with probability p (success), or zero with probability $1-p$ (failure) on date 1. Similarly, if firm D invests I^D on date 0, it will receive a cash flow of $K^D I^D$ with probability p, or zero with probability $(1-p)$ on date 1 (the two projects are independent). That is, the production function of both firms is linear, with K^T being the proportionality constant. We also allow that the firm may have a technology with decreasing returns to scale. We show in the Appendix that our model's implications about the resource allocation

[5]We opt for a pyramidal structure of control, but the results will be the same if we use a horizontal structure.

in business groups essentially hold in this extension of a more general production function.

The timing of the model is shown in Figure 1.

[Insert Figure 1 about here]

To introduce moral hazard, we assume that the probability of success (of each project) depends on the entrepreneur's efforts. Therefore, if the entrepreneur behaves (exerts effort), the probability of success is p_H and there are no private benefits. If the entrepreneur misbehaves, the probability of success is $p_L < p_H = p_L + \Delta_p$ and the private benefits are B^U (B^D) per unit of investment in firm U (D).[6] That being said, as long as the projects are funded, the entrepreneur can work on either one or both, or cheat on both. Only projects with a probability p_H of success are considered socially desirable. In other words, p_L is assumed such that, if the entrepreneur misbehaves, the expected NPV (social surplus) per unit of investment is negative, even if private benefits are considered.

$$\begin{aligned} p_H K^U &> 1, \\ p_L K^U + B^U &< 1, \\ p_H K^D &> 1, \\ p_L K^D + B^D \div \beta &< 1. \end{aligned} \tag{A1}$$

To achieve a finite level of optimum investment, we need to make an additional assumption about the productivity of investment and the extent of moral hazard (regarding pledgeable income). Following Tirole (2006), the expected NPV per unit of investment is lower than the per-unit agency cost related to the entrepreneur's misbehavior (i.e., the expected minimal income per unit of investment that is incentive compatible):

[6]Note that we are assuming that private benefits are asset specific, not human specific. Although we recognize that business groups can transfer human resources across affiliates, in our model changing the entrepreneur does not change the private benefits associated with each firm in the group.

$$p_H\left(K^U - \frac{B^U}{\Delta_p}\right) < 1,$$
$$p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right) < 1. \tag{A2}$$

Therefore, there is a limit to the value that firms can raise from external investors, imposing a specific investment level, even though infinite levels of investment are optimal under no moral hazard. Assumption A2 is key to our model, as it implies financial constraints at the firm level: for each unit of investment, the income that can be pledged to outside investors, that is, the expected cash flow less the expected minimal income that ensures the entrepreneur will behave, is less than one, and firms must therefore supplement this amount with internal resources to finance this unit of investment. In other words, the amount that firms can raise in external capital markets and the level of investment depend partly on the internal resources available.

This dependence on internal wealth is at the core of the investment models with moral hazard developed by Tirole (2006). For business groups, it has an important consequence: the amount available to the internal capital market is limited to internal wealth. The maximum amount that the entrepreneur can shift from one firm to another is the internal resources available in the first firm.

Continuing, on date 0, firm U (D) has liquid assets (i.e., cash holdings) of A^U (A^D), and there is an internal capital market in which firms U and D can transfer resources between them on date 0 in exchange for an income on date 1. We denote by $(1-\alpha^T)$, with $T \in \{U, D\}$, the (observable) fraction of cash on date 0 that is transferred from one firm to another. An upper bound on the internal transfers (perhaps as a result of legal and statutory limits) is imposed,[7] requiring that $\alpha^T \in [\alpha, 1]$, with $0 < \alpha < 1$.

[7] Actually, assumption A2 will constrain α^T to be greater or equal to zero. We require α^T to be strictly positive so both firms invest. In the Appendix, we comment on this assumption and show that our predictions do not change.

We also assume that business groups use direct loans to make internal transfers across affiliated firms. A direct loan is a common mechanism for allocating resources within business groups ((Gopalan et al., 2007), Buchuk et al., 2014). As Buchuk et al. (2014) point out, the widespread existence of preemptive rights is the main reason why direct loans (internal debt) are often more convenient than internal equity (cross-ownership) as a way of transferring resources within a business group. In part, this is because preemptive rights give current shareholders the right to buy new shares issued by the firm, protecting them against the dilution of control of their shares.

We now assume that date 1 income from internal transfers cannot be contracted out of the business group. In other words, the lending firm cannot pledge this income to outside investors. For simplicity, interest rates are set to zero (no time discount). Under these conditions, the borrower, say, U, needs to promise an amount $(1-\alpha^D)A^D \div p_H$ on date 1, in the case of success, in exchange for a loan of $(1-\alpha^D)A^D$ on date 0 (we opt for a conditional debt contract between firms).

Because firms U and D are *legally independent*, we assume no cross-pledging, where one firm could potentially pledge another affiliate's income to external investors (lenders). This means that each group firm has its own budget constraints. Had we allowed cross-pledging, the entire business group would behave as if it had a single joint budget constraint, as in a diversified conglomerate, and the insurance and tunneling effects of Samphantharak (2006) would emerge.[8] Finally, we assume that the lender sector is competitive. Therefore, by having control over both firms, the entrepreneur will offer a contract to outside investors as follows:

- Firms' income in each state of the world (success, S, or failure, F): $R_S^T \geq 0$ and $R_F^T \geq 0$, with $T \in \{U, D\}$; that is, both the lender's and the borrower's limited liability imply that firms will receive zero in the case of failure.

[8]One can also argue that business groups could raise more resources than a comparable portfolio of standalone firms, due to coinsurance effects, for example. As pointed out by Berger and Ofek (1995) and Scharfstein and Stein (2000) for diversified conglomerates, these effects are of trivial importance.

- Each firm's level of investment: $I^T \geq 0$, with $T \in \{U, D\}$.
- Internal transfers from one firm to another: $(1-\alpha^T)A^T$, with $T \in \{U, D\}$.

The contract will solve the following problem (for details, see the Appendix):

$$\max_{\{R_S^T, R_F^T, I^T, \alpha^T\}} p_H \left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) + (1-p_H)R_F^U + p_H \left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H} \right) + (1-p_H)\beta R_F^D,$$

subject to four constraints that are binding at the optimal solution. The "investor rationality" constraints, IR^U and IR^D, require that, on average, outside investors get back their investment:

$$p_H \left(K^U I^U - R_S^U \right) - (1-p_H)R_F^U \geq I^U - \alpha^U A^U - (1-\alpha^D)A^D, \qquad (IR^U)$$

$$p_H \left(K^D I^D - R_S^D \right) - (1-p_H)R_F^D \geq I^D - \alpha^D A^D - (1-\alpha^U)A^U, \qquad (IR^D)$$

and the incentive compatibility constraints, IC^U and IC^D, ensure that the entrepreneur will choose to behave well in both projects:

$$\Delta_p \left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) \geq B^U I^U, \qquad (IC^U)$$

$$\Delta_p \left(\beta(R_S^D - R_F^D) + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H} \right) \geq B^D I^D. \qquad (IC^D)$$

The non-negativity and internal transfer limitation constraints are expressed as follows:

$$R_S^U \geq 0,\ R_F^U \geq 0,\ R_S^D \geq 0,\ R_F^D \geq 0,\ I^U \geq 0,\ I^D \geq 0,$$

$$\alpha^U \in [\alpha, 1]\,,\ \alpha^D \in [\alpha, 1]\,.$$

Because the lender sector is competitive, the firms will earn the entire surplus. Under our assumptions about the productivity of investments and moral hazard in (A1) and (A2) and the limits of internal transfers, it is optimal that both firms invest: $I^U > 0$ and $I^D > 0$.

The investor's rationality constraints are binding; otherwise, firms could increase their payoffs without violating the incentive compatibility constraints. To show that the incentive compatibility constraints are also binding at the optimum, suppose that (IC^U) is not binding (the same applies to (IC^D)). Then, R_S^U, R_F^U, and I^U could be increased as long as the difference $R_S^U - R_F^U$ is constant and the increase in the payoffs is limited to $(p_H K^U - 1)$ times the increase in I^U. These changes will increase the value of the objective function without violating the investors' rationality constraint, so this solution cannot be optimal.

With strictly positive investment, the incentive compatibility constraint (IC^U) implies that $R_S^U > R_F^U \geq 0$ (again, the same applies to firm D). Because the entrepreneur is risk neutral and will earn the entire social surplus of the investment, it is best for the entrepreneur to set the firm payoffs at a level that maximizes the pledgeable income. From investors' rationality constraint (IR^U), the pledgeable income is given by

$$p_H K^U I^U - p_H(R_S^U - R_F^U) - R_F^U.$$

Maintaining the difference $R_S^U - R_F^U$ to satisfy the incentive compatibility constraint and setting $R_F^U = 0$ maximizes the pledgeable income.[9] Under these conditions, the incentive compatibility constraints can be used to determine the payoffs in the case of success:

$$R_S^U = \frac{B^U I^U}{\Delta_p} + \frac{(1-\beta)(1-\alpha^D)A^D}{p_H}, \tag{1}$$

[9] In the Appendix, we provide proof that, at the optimum, $R_F^U = 0$ and $R_F^D = 0$, using the Karush–Kuhn–Tucker multipliers.

$$R_S^D = \frac{B^D I^D}{\beta \Delta_p} - \frac{(1-\beta)(1-\alpha^U)A^U}{\beta p_H}. \tag{2}$$

The investors' rationality constraints determine the level of investment of each firm (after replacing R_S^U and R_S^D by (1) and (2)), as follows:

$$I^U = \frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} = M^U \times \left(\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D\right), \tag{3}$$

$$I^D = \frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\beta\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} = M^D \times \left(\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U\right). \tag{4}$$

In (3) and (4), respectively, M^U and M^D are the equity multipliers, where equity means the entrepreneur's wealth ($A^U + \beta A^D$), split between firms by internal transfers. One can see that, under assumptions (A1) and (A2), both multipliers are greater than one but finite. They are finite because, under assumption (A2), the minimal income that is incentive compatible increases faster than the NPV when investment is increased. Therefore, investors' rationality constraints bind with finite levels of investments. In short, moral hazard implies limits to the investment level, reducing the entrepreneur's utility.

It is worth noting that, if firms U and D were standalone entities, their equity multipliers would be the same as in (3) and (4), respectively. However, in this case, each firm can only rely on the entrepreneur's wealth. Internal capital markets in business groups can transfer the entrepreneur's wealth across group firms. Hence, with the appropriate incentives, the entrepreneur can increase the total output (over what it would be if the group firms were standalone entities).

The entrepreneur will earn the surplus from investment according to the number of shares owned in each firm and will thus benefit from higher multipliers. Taking the partial derivatives of the multipliers with respect to the exogenous parameters, we have:

- Multipliers increase with p_H and Δ_p. All else being equal, the higher p_H (Δ_p), the

greater the income that can be pledged to outside investors and the lower the minimal income that makes the entrepreneur behave.

- The term M^U (M^D) increases with K^U (K^D). All else being equal, more productive investment attracts more external finance.

- The term M^U (M^D) decreases with B^U (B^D). The minimal income that the entrepreneur needs to behave increases with private benefits, reducing the pledgeable income.

- The term M^D (but not M^U) increases with β. All else being equal, the higher the entrepreneur's cash flow rights in firm D, the lower the minimal income the entrepreneur needs to behave and, consequently, the higher firm D's pledgeable income.[10]

What remains to be determined are the internal transfers between firms, α^U and α^D. To show how the internal capital market works, the entrepreneur's problem is rewritten using the optimal values of the endogenous variables, except α^U and α^D. The entrepreneur's expected total income equals the expected NPV of the investment in firm U plus a fraction β of the expected NPV of the investment in firm D plus the entrepreneur's initial wealth, $A^U + \beta A^D$, as follows:

$$(p_H K^U - 1)I^U + \beta(p_H K^D - 1)I^D + A^U + \beta A^D. \tag{5}$$

Substituting (3) and (4) into I^U and I^D, respectively, the entrepreneur's objective function becomes:

[10] Our assumption is that the entrepreneur has all the cash flow rights in firm U. If we assume that the cash flow rights are of size β^U, the equity multiplier of this firm, M^U, will also increase with β^U.

$$F(\alpha^U, \alpha^D) = (p_H K^U - 1)\frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} + (p_H K^D - 1)\frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} + A^U + \beta A^D. \tag{6}$$

Next, it is possible to determine how the entrepreneur's expected total income changes when α^U or α^D increases:

$$\frac{\partial F(\alpha^U, \alpha^D)}{\partial \alpha^U} = \frac{(p_H K^U - 1)A^U}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} - \frac{(p_H K^D - 1)A^U}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]}, \tag{7}$$

$$\frac{\partial F(\alpha^U, \alpha^D)}{\partial \alpha^D} = \frac{(p_H K^D - 1)\beta A^D}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} - \frac{(p_H K^U - 1)\beta A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]}. \tag{8}$$

Note that the partial derivatives depend only on the exogenous parameters and, if (7) is positive [negative] (zero), then (8) is negative [positive] (zero), and vice versa. Thus, there are three possible alternatives of internal transfers in business groups that we discuss in turn.

A Internal transfers from D to U

Internal transfers from D to U occur if and only if

$$B^D(p_H K^U - 1) > B^U \beta (p_H K^D - 1).$$

If this condition holds, (7) is positive, (8) is negative, and the entrepreneur's expected total income increases with α^U and decreases with α^D. Three factors can contribute to this result: (a) The investment productivity of firm U, K^U, is higher than that of firm D, K^D; (b) there are fewer private benefits associated with firm U's investment, B^U, than with firm D's investment, B^D—that is, all else being equal, the minimal income that motivates the entrepreneur to behave is lower and, therefore, pledgeable income is higher in firm U vis-à-vis

firm D—and (c) entrepreneur cash flow rights in firm D, β, are low enough to distort the socially efficient capital allocation.[11]

As the entrepreneur's expected total income increases (decreases) with α^U (α^D), the internal transfer goes from firm D to firm U, up to the upper bond of internal transfers in which $\alpha^U = 1$ and $\alpha^D = \alpha$. In this case, the sensitivities of the firm's investment to its cash flow and to the other firm's cash flow are[12]:

$$\begin{aligned}
\frac{\partial I^U}{\partial A^U} &= M^U > 0, \\
\frac{\partial I^U}{\partial A^D} &= M^U \times \beta(1-\alpha) > 0, \\
\frac{\partial I^D}{\partial A^U} &= 0, \\
\frac{\partial I^D}{\partial A^D} &= M^D \times \beta\alpha > 0.
\end{aligned}$$

Because of the unidirectionality of internal transfers, firm U's investment increases with its cash flow and with the other firm's cash flow, and firm D's investment increases with its cash flow and is insensitive to the other firm's. In this case, the business group's resources flow toward firm U, and the investment in firm D is proportional to its cash flow and occurs only because there are limits to internal transfers.

Finally, following Tirole (2006), the sensitivity of investment to the firm's cash flow (and to the other firm's cash flow, when applicable) is reduced with the private benefits. This happens because of the negative effect of private benefits on pledgeable income and, consequently, on the equity multiplier. Therefore, in our model, firms with low agency costs will exhibit greater investment-cash flow sensitivity.

[11] Social efficiency refers to the allocation that provides the higher expected NPV. In our setting, the NPV of an investment depends not only on its productivity, but also on its capacity to attract financing. The β is a (inverse) measure of control leverage, the difference between voting and cash flow rights.

[12] In our static one-period model, A^U and A^D can be regarded as both a flow (cash flows from existing assets) and a stock (cash holdings). We use comparative statics on A^U and A^D to derive our investment–cash flow sensitivities. To justify this, we resort to DeMarzo and Fishman (2007).

B Internal transfers from U to D

Internal transfers from U to D occur if and only if

$$B^D(p_H K^U - 1) < B^U \beta(p_H K^D - 1).$$

If this condition is met, (7) is negative and (8) is positive, so the entrepreneur's expected total income decreases with α^U and increases with α^D. Again, three factors can contribute to this result: (a) The investment productivity of firm D, K^D, is higher than that of firm U, K^U; (b) there are fewer private benefits associated with firm D's investment, B^D, than with firm U's investment, B^U—that is, all else being equal, the minimal income that encourages the entrepreneur to behave is lower and, therefore, the pledgeable income is higher in firm D vis-à-vis firm U—and (c) the entrepreneur's cash flow rights in firm D, β, are high enough not to distort the socially efficient capital allocation.

As the entrepreneur expected total income decreases (increases) with α^U (α^D), the internal transfer goes from firm U to firm D, up to the upper bond on internal transfers in which $\alpha^U = \alpha$ and $\alpha^D = 1$. In this case, the sensitivities of a firm's investment to its cash flow and to the other firm's cash flow are

$$\begin{aligned}
\frac{\partial I^U}{\partial A^U} &= M^U \times \alpha > 0, \\
\frac{\partial I^U}{\partial A^D} &= 0, \\
\frac{\partial I^D}{\partial A^U} &= M^D \times (1-\alpha) > 0, \\
\frac{\partial I^D}{\partial A^D} &= M^D \times \beta > 0.
\end{aligned}$$

Firm D's investment increases both with its cash flow and with the other firm's cash flow. Firm U's investment increases with its cash flow and is insensitive to the other firm's cash flow. Now, business groups' resources flow toward firm D, and the investment in firm U is

proportional to its cash flow and occurs only because there are limits to internal transfers.

As before, the sensitivity of investment to the firm's cash flow (and to the other firm's cash flow, when applicable) of both firms decreases with private benefits. This happens because of the negative effect of private benefits on pledgeable income and, thus, on the equity multiplier. Again, firms with low agency costs will exhibit greater investment–cash flow sensitivity.

C No internal capital market

There is no internal capital market if and only if

$$B^D(p_H K^U - 1) = B^U \beta (p_H K^D - 1).$$

If this condition is met, (7) and (8) are equal to zero, and the entrepreneur's expected total income does not depend on α^U or α^D. This independence of the entrepreneur's income from α^U and α^D can occur if, for example, the private benefits and the NPV per unit of investment (from the entrepreneur's perspective) are very similar across firms.

As the entrepreneur's expected total income does not depend on α^U or α^D, the internal transfers are undetermined; that is, any admissible values of α^U and α^D are optimal. We assume that, under these circumstances, the entrepreneur will opt for the simplest contract where there is no transfer across firms ($\alpha^U = \alpha^D = 1$). Consequently, the investment–cash flow sensitivities are

$$\begin{aligned}
\frac{\partial I^U}{\partial A^U} &= M^U > 0, \\
\frac{\partial I^U}{\partial A^D} &= 0, \\
\frac{\partial I^D}{\partial A^U} &= 0, \\
\frac{\partial I^D}{\partial A^D} &= M^D \times \beta > 0.
\end{aligned}$$

The investments in firms U and D increase with firm cash flows and are insensitive to the other's firm's cash flow. Thus, without internal transfers, the investment in each firm is proportional to the entrepreneur's cash flow in that firm, with the constant of proportionality equal to the equity multiplier. Finally, the sensitivity of investment to cash flow for both firms decreases with private benefits. Again, the investments of companies with low agency costs will be more sensitive to their cash flow.

[Insert Figure 2 about here]

To better illustrate how differences in investment productivity (K^U and K^D), private benefits (B^U and B^D), and entrepreneur's cash flow rights (β) affect the allocation of resources in our model of business groups' internal capital markets, we plot the outcomes of this process, that is, which firm (U or D) will receive internal resources (alternatives A to C above) as we change the values of these variables. We start from a base case in which we set the variables as follows: $p_H = 0.7$, $K^U = 1.5$, $K^D = 1.6$, $B^U = 0.048$, $B^D = 0.06$, and $\beta = 0.75$. In this base case, firm D is more productive, but its investment project has higher private benefits compared to firm U's project. Moreover, the entrepreneur's cash flow rights are higher in firm U (100%) than in firm D (75%). Despite these limitations, in this base case, the resources of this internal capital market should flow toward firm D (alternative B), as its higher productivity more than compensates for the bigger agency problems it faces.

In Figure 2, we let B^U and B^D be fixed at their base case values and we vary K^U (horizontal axis), K^D (vertical axis), and β (panels). All else being equal, an increase in the productivity of a firm tends to favor it as a receiver of resources in the internal capital market. For example, an increase in firm U's productivity (horizontal movement) makes this firm a better candidate to receive internal resources. The same applies to firm D (vertical movement). Regarding the entrepreneur's cash flow rights in firm D (β), the higher the β, the smaller the difference between the productivity of firm D and that of firm U must be so that the former is the receiver of resources in the internal capital market. In the hypothetical

case in which both firms' productivity follows a uniform distribution in the intervals in the graphs, the probability that firm D (U) will be the receiver of internal resources corresponds to the fraction of the total dark gray area in the figure. As shown, the more the entrepreneur's cash flow rights in firm D increase (from the left to the right panel), the higher (lower) the likelihood that firm D (U) will receive resources in the internal capital market.

In Figure 3, we let K^U and K^D be fixed at their base case values and we vary B^U (horizontal axis), B^D (vertical axis), and β (panels). All else being equal, an increase in private benefits in one firm tends to favor other firms in the internal capital market as potential receivers of resources. For example, an increase in firm D's private benefits (vertical movement) makes this firm a better candidate to receive internal resources. The opposite occurs if firm U's private benefits are increased (horizontal movement). The higher the entrepreneur's cash flow rights in firm D, the higher the difference between the private benefits of firms D and U must be so that the latter is the target of resources in the internal capital market. In the hypothetical case in which both firms' private benefits follow a uniform distribution in the intervals in the graphs, the probability that firm D (U) will be the receiver of internal resources corresponds to the fraction of the total dark gray area. As shown, as the entrepreneur's cash flow rights in firm D increase (from the left to the right panel), the higher (lower) the likelihood that firm D (U) will be the receiver of resources in the internal capital market.

[Insert Figure 3 about here]

In the next section, we discuss the key results of our model and relate them to the literature.

4 Discussion and propositions

According to our model, the direction of resources inside business groups depends on three factors: (a) the investments' productivity differences, (b) the entrepreneur's cash flow rights differences, and (c) the differences in private benefits, or, in other words, in pledgeable income. The first two factors have been extensively studied in the internal capital market literature, whereas the latter, as shown below, has not.

The first factor, investment productivity differences, is related to the allocation efficiency in internal capital markets. For example, Williamson (1975) argues that, in a multidivisional firm, the top management team (CEO and C-level executives) can perform a capital market function – assigning cash flows to high NPV projects. Stein (1997) develops a model in which headquarters, with the proper incentives and control rights to supervise project outcomes, engage in a winner-picking strategy, allocating scarce resources to projects with higher returns.

Empirical results, however, raise doubts about the allocation efficiency of internal capital markets in conglomerates. Shin and Stulz (1998), Rajan et al. (2000), Billett and Mauer (2003), and Ozbas and Scharfstein (2010) present evidence that internal capital markets tend to allocate resources inefficiently, investing too much (or too little) in divisions with few (or many) investment opportunities, so-called socialist cross-subsidization.[13] Campello (2002) examines internal transfers across small affiliate banks of multi-bank holding companies. The author's findings are consistent with the inefficient (efficient) cross-subsidization hypothesis in constrained (unconstrained) bank holding companies. More recently, studies have suggested that the allocation efficiency of internal capital markets improves during financial crises (Kuppuswamy and Villalonga, 2016) and recessions (Hovakimian, 2011) and when there is external capital market distress (Matvos and Seru, 2014), that is, when finan-

[13] Rajan et al. (2000) and Scharfstein and Stein (2000) develop models that imply inefficient cross-subsidization of this type in internal capital markets. The term *socialist* cross-subsidization was introduced by Stein (2003).

cial constraints are more likely to be binding, and hence the winner-picking strategy is more valuable.[14]

Empirical studies also report mixed results in allocation efficiency in business groups. Shin and Park (1999) present evidence that Korean business groups (chaebols) better insulate the investment of high-growth firms from group-level financing constraints, which is consistent with the efficiency hypothesis outlined by Shin and Stulz (1998). Shin and Park (1999) also show that capital expenditures (as a fraction of total assets) do not differ between high- and low-growth chaebols' firms, whereas this is not so for non-chaebols' firms. This result is consistent with the socialist cross-subsidization view. Lee et al. (2009) demonstrate that, before the 1997 Asian crisis, chaebols' firms with high-growth opportunities took more advantage of cross-subsidization than other firms with poor opportunities in the same group; however, the same was not true after the crisis.

In addition, Almeida et al. (2015) present evidence suggesting that chaebols engaged in winner-picking strategies in the aftermath of the Asian crisis. Gopalan et al. (2007), in a study of Indian business groups, also show that net intra-group loans are insensitive to growth opportunities and that firms receiving intra-group loans underperform benchmarks in the two-year period following a loan, suggesting that efficiency is not the primary goal of Indian internal capital markets.

The second factor, the entrepreneur's cash flow rights in affiliated firms, is related to the likelihood of the entrepreneur diverting wealth from firms he or she controls without holding proportional cash flow rights, known as *private benefits of control*. As Morck et al. (2005) emphasize, by separating cash flow and voting rights, pyramidal structures creates the same divergence of interest problems as dispersed ownership. This divergence, therefore, can lead to inefficient investment in firms in which a controlling owner has small cash flow rights. In our model, as we assume that the entrepreneur always controls firm D, the lower the entrepreneur's cash flow rights in firm D (low values of β), the wider the wedge between

[14] Gopalan and Xie (2011) present mixed results about the efficiency of internal capital markets during periods of *unexpected* industry distress.

cash flow and voting rights.

The greater the wedge between cash flow and voting rights (control) in firm D, the higher the likelihood that resources inside the group flow toward firm U, even if firm D has better investment opportunities ($K^D > K^U$). Consequently, it is possible for the entrepreneur to externalize most of the costs related to value-destroying investments, creating economic incentives to divert corporate wealth at the expense of outside investors, that is, tunneling (Johnson et al., 2000). This divergence of interests between the entrepreneur (inside shareholder) and the outside investor (minority shareholders) arises because, from the entrepreneur's point of view, a unit of investment in firm U (D) has an expected value of $p_H K^U - 1$ ($\beta(p_H K^D - 1)$). Therefore, for low values of β, investment inefficiency in business groups and outside investor losses are more likely to be observed.[15]

Empirical studies have tried to identify the tunneling effect with inconclusive results. For example, examining Indian business groups, Bertrand et al. (2002) present evidence of tunneling whereas Siegel and Choudhury (2012) show that internal transfers are driven by business strategies that differ remarkably across business group firms and standalone firms. The analysis of Gopalan et al. (2007)of intra-group loans in Indian business groups shows that net intra-group loans are positively related to insider cash flow rights and are primarily used to provide finance for impaired firms, with no evidence of tunneling. Buchuk et al. (2014) present similar evidence for Chilean firms, suggesting that, although a conclusion of tunneling could not be completely ruled out, intra-group loans are typically used to reduce financial constraints and increase investment.

Relative to the third factor, to the best of our knowledge, we still do not have a theory to explain how do private benefits influence the direction of resource allocation in the internal capital market. In a business group, each firm is a legally independent entity with direct access to the external capital market. To access this market, each group-affiliated firm can rely only on its merits and its pledgeable income. As equation (6) shows, the surplus of

[15]What really matters is the relative size of the entrepreneur cash flow rights in firm D and U, captured by β in our model.

an investment depends on the interaction between its marginal expected net present value (productivity) and its equity multiplier (pledgeable income). Therefore, the entrepreneur will direct resources to the firm with the higher product (in her eyes) of investment productivity and pledgeable income. That is, pledgeable income matters in financing decisions in both internal and external capital markets.

Our model implies that pledgeable income is negatively related to private benefits (B^U and B^D). The greater the private benefits, the higher the minimal income necessary for the entrepreneur to behave, and, thus, the lower the pledgeable income. Consequently, low levels of private benefits increase the likelihood of financing new investment in both internal and external capital markets.

The same reasoning applies to any factor affecting pledgeable income. The entrepreneur's cash flow rights in firm D, β, also impact the firm's pledgeable income. The higher β is, the lower the minimal income that the entrepreneur needs to behave well in firm D, and the higher the pledgeable income, the equity multiplier, and the likelihood of accessing funding in the internal capital market.[16] Similarly, as the investment productivity (K^U and K^D) increases, so do the social surplus, the pledgeable income, and the equity multiplier. Thus, as investment productivity grows, so do the odds of obtaining internal resources from other business group firms, as well as external finance.

To show that any factor affecting the firm's ability to raise external finance also affect the firm's likelihood of obtaining financing in the internal capital market, assume that one unit of investment in firm U (D) requires raising $\tau^U \geq 1$ ($\tau^D \geq 1$) units of internal or external money. We can think of τ^T as a proxy for factors that reduce the firm's ability to finance its projects, including low pledgeable assets (collateral), a high probability of risk shifting, and high levels of asymmetric information . Assuming that the investment in both firms is still profitable, we can show that

[16] Again, if we assume that the entrepreneur's cash flow rights in firm U is β^U, this same effect will be present in firm U.

$$I^U = \frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[\tau^U - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} = M'^U \times \left(\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D\right), \tag{3'}$$

$$I^D = \frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\beta\left[\tau^D - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} = M'^D \times \left(\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U\right). \tag{4'}$$

Hence, under financial constraints, the equity multiplier of both firms is reduced; that is, if $\tau^U > 1$, then $M'^U < M^U$, and if $\tau^D > 1$, then $M'^D < M^D$. This is the result of a reduction in the pledgeable income of the firms and implies a lower level of investment. The direction of resources inside the business group will now depend on the following inequality:

$$B^D\left(p_H K^U - \tau^U\right) \gtreqless B^U\beta\left(p_H K^D - \tau^D\right).$$

All else being equal, the higher the τ^U (τ^D), the lower the chance that internal resources will flow from firm D (U) to firm U (D). In other words, the same factors that limit a firm's access to external finance also reduce the likelihood that the same firm will receive resources in the internal capital market. As far as we know, this is a novel prediction, shedding new light on our understanding of the formation and functioning of business groups.

The prediction that productivity (growth opportunities) and pledgeable income jointly determine the direction of resources in the internal capital market explains, in part, the evidence of socialist cross-subsidization in business groups. For example, if there is a low correlation between productivity and pledgeable income and the latter is more volatile than the former, our model predicts that pledgeable income will be the most important factor in explaining resource allocation within a business group. This model outcome implies that the omission of pledgeable income from the analysis of allocation efficiency in business groups' internal capital markets can produce a conclusion biased toward the socialist cross-subsidization hypothesis.

The omitted variable bias is especially worrisome if the correlation between productivity

and pledgeable income is negative, where a highly productive asset can increase the concerns about private benefits, risk shifting, and low collateral. This potential bias could explain prior empirical results (Shin and Park, 1999; Lee et al., 2009;Almeida et al., 2015). For example, assuming that productivity and pledgeable income are highly correlated during recessions and financial crises, our model could explain the results of Almeida et al. (2015), that Korean chaebol groups engaged in winner-picking strategies in the aftermath of the 1997 Asian crisis.

The main implication of our model is, therefore, that the same factors that limit companies' access to external finance also reduce the chance of obtaining internal resources in business groups. Firms with high levels of pledgeable income (and thus easy access to external finance) will be more likely to benefit from resource allocation within a business group. In other words, internal capital markets tend to support the financially strong firms in a group, just as outside lenders would, reproducing the same financial constraints that plague external financial markets. This implication can be restated as in the following proposition.

Proposition 1. The likelihood of obtaining resources in the internal capital market increases with productivity, pledgeable income, and the entrepreneur's cash flow rights.

In other words, according to our model, receivers of intra-group loans tend to be those group firms that (relative to other firms in the same group) have high productivity, low private benefits, high cash flow rights of controlling shareholders, and high asset tangibility, for example. Except for the productivity and cash flow rights factors, this is a novel hypothesis, and it is discussed at length in the final part of the previous section. If the above proposition is true, financially strong firms in business groups will be able to raise more resources and, consequently, will invest more than their standalone counterparts. The contrary occurs with financially weak firms. These are more likely to be lenders, supporting the internal capital market and relying on only a fraction of their wealth to finance their investments. Thus, financially weak firms will have fewer resources available and tend to invest less than their standalone counterparts, which do not have related firms to finance. This reasoning leads to

the following proposition.

Proposition 2. All else being equal, financially strong (weak) firms in the business group tend to invest more (less) than their standalone counterparts, because these firms tend to benefit from (support) internal capital markets.

If true, this hypothesis raises questions about the effectiveness of internal capital markets in overcoming external capital markets' failures, as hypothesized by Khanna and Palepu (2000) and Khanna and Yafeh (2007). Given that productivity and pledgeable income have a positive effect on the likelihood of getting resources from internal capital markets, the factors that improve a firm's ability to get external finance, such as asset tangibility, also increase the likelihood of internal financing in business groups. Buchuk et al. (2014) show that, in Chilean business groups, capital-intensive (a proxy for pledgeable income) and small firms are more likely to receive intra-group loans, which support our hypothesis. In contrast, Gopalan et al. (2007)'s results for Indian business groups show that net intra-group loans decrease with asset tangibility and are insensitive to growth opportunities. In other words, our hypothesis fits well in Chilean firms but not in Indian firms suggesting that institutions are relevant.

According to Buchuk et al. (2014), three features of the Chilean regulation that stand out in comparison to other markets. First, Chilean law requires the full disclosure of all related loans (in great detail), allowing investors to easily identify intra-group loans. Second, Chilean law requires that such loans be made at the prevailing market interest rate, whereas in India Gopalan et al. (2007) show that more than 80% of intra-group loans have no interest obligation at all. Finally, in Chile, transactions between related parties require approval by a board committee presided by an independent director.

Our model captures some of these institutional features, postulating that (a) the contract with external investors should specify all internal transfers between firms in the business group, (b) the interest rate on intra-group loans is the same as in competitive external markets (to simplify, this rate is assumed to be zero), and (c) although there is room for

minority shareholder expropriation, productivity and pledgeable income (financial capacity) drive resources allocation in the internal capital market, and these factors are likely to satisfy the requirements of an independent director.

The next propositions are related to the sensitivity of investment to the focal firm's cash flow and other group-affiliated firms' cash flow.

Proposition 3. Investment–cash flow sensitivity is positive and increases with pledgeable income (e.g., firms with high productivity, low private benefits, high controlling shareholder cash flow rights, and high asset tangibility will exhibit greater investment–cash flow sensitivity).

This proposition is not particularly new. Theoretically, it has been derived from Tirole's (2006) models. Empirically, several authors, including Fazzari et al. (1988), Hoshi et al. (1991), and Kaplan and Zingales (1997), have documented the positive effect of cash flow on firm investment. The cross-sectional variation in the sensitivity of investment to cash flow, however, has been the subject of debate in the corporate finance literature. Although Fazzari et al. (1988) present evidence that investment–cash flow sensitivity increases with the degree of financial constraint, Kaplan and Zingales (1997) challenge this view, both theoretically and empirically.

We suggest that investment becomes less sensitive to cash flow with the degree of financial constraints, in accordance with the work of Kaplan and Zingales (1997). For example, if firms with few private benefits and considerable asset tangibility are less financially constrained, then our model implies that their investment–cash flow sensitivity will be high. This hypothesis is also consistent with the results of Almeida and Campello (2007). These authors propose that tangible (pledgeable) assets support more borrowing, allowing for further investment in tangible assets, giving rise to a credit multiplier. They show that the sensitivity of investment to cash flow increases with asset tangibility for financially constrained firms, as suggested by the credit multiplier rationale.

In the business group literature, to the best of our knowledge, there is no evidence on

how financial and agency factors (asset tangibility and entrepreneurs' private benefits) jointly affect the investment–cash flow sensitivity of group-affiliated firms. We propose a theoretical model that explicitly recognizes the role of group firms' pledgeable income in internal and external capital markets, calling for future empirical evidence.

Proposition 4. The sensitivity of investment to other group-affiliated firms' cash flow is positive (null) for receivers (providers) of intra-group loans and increases with their level of pledgeable income.

According to our model, the sensitivity of investment to other group firms' cash flow is the outcome of an active internal capital market in business groups. Lamont (1997), Shin and Stulz (1998), Shin and Park (1999), and Lee et al. (2009) use this logic to motivate their empirical analyses and to interpret their results.[17] They report that the cash flow of other segments (firms) in the same conglomerate (business group) positively affects firm investment. This evidence supports the internal capital markets hypothesis.

As far as we know, however, no study differentiates between the investment–cash flow sensitivity of receivers and providers of capital within business groups. Our model explicitly makes this singular distinction and predicts that only the receivers' investment is positively affected by the cash flow of other firms in the business group.

The extent of the effect of other group firms' cash flow on investment depends on the receivers' equity multiplier or, from another perspective, pledgeable income. Receivers with high levels of pledgeable income can leverage internal wealth to a greater degree (higher multiplier), and their investment therefore responds more strongly to other firms' cash flow than the investment of receivers with low pledgeable income. Thus, our model suggests that the investment of receivers who have high productivity, low private benefits, high cash flow rights of controlling shareholders, and high asset tangibility will be more sensitive to other group firms' cash flow.

[17] Lamont (1997) and Shin and Stulz (1998) address conglomerates, whereas Shin and Park (1999) and Lee et al. (2009) address business groups.

Moreover, the corporate finance literature presents mixed evidence on the effect of productivity on the sensitivity of investment to other group firms' cash flow. On the one hand, the results of Shin and Stulz (1998) suggest that the sensitivity of a segment's investment to the cash flow of other segments does not depend on whether its investment opportunities are better than those of the other segments. For business groups, Shin and Park (1999) suggest a lower investment sensitivity to other group firms' cash flow for firms with strong growth opportunities. On the other hand, Lee et al. (2009) find the opposite in the period preceding the 1997 Asian crisis. Our model also accounts for the sensitivity of investment to other firms' cash flow variations with private benefits, controlling shareholder cash flow rights, or asset tangibility, opening a new avenue for future research

5 Concluding remarks

We provide a new rationale for investment in business groups subject to moral hazard to answer two related questions: (1) How do business groups allocate resources in internal capital markets? (2) And do the internal capital markets alleviate the financial constraints of affiliate firms that have limited access to external finance?

To answer the first question, our model suggests that productivity and pledgeable income jointly determine the allocation of resources in business groups' internal capital markets. That is, funds within groups tend to flow in the direction of firms with high productivity and high pledgeable income. This means that, if productivity varies little relative to pledgeable income across firms within a group, pledgeable income will be the key driver of resources.

To answer the second question, our model predicts that internal capital markets in business groups tend to favor financially strong firms over financially weak firms. This result casts doubt on the ability of internal capital markets to alleviate the financial constraints of group firms that have limited access to external finance, as hypothesized by Khanna and Palepu (2000) and Khanna and Yafeh (2007).

Our model's primary predictions are consistent with evidence on intra-group loans in Chilean business groups (Buchuk et al., 2014), but inconsistent with the results of Gopalan et al. (2007) for Indian business groups. Institutions, therefore, could play a major role in explaining cross-country variations in business groups' financial allocation. In several aspects, our model assumptions resemble those of the Chilean institutional environment. Our model can also be used to gain a better understanding of the efficiency of internal capital markets in business groups (Shin and Park, 1999; Gopalan et al., 2007; Almeida et al., 2015). We propose that efficiency in capital allocation is driven by productivity *and* pledgeable income. Therefore, our model could explain in terms of efficiency evidence suggesting socialist cross-subsidization in business groups.

We believe that examination of the welfare effects of internal capital markets in business groups provides a unique contribution to the corporate finance literature in emerging economies, where capital markets are less developed and business groups are ubiquitous. Along with the insights of Almeida and Wolfenzon (2006b,a), our study's testable implications can inspire further theoretical and empirical work aimed at gaining a better understanding of the equilibrium effects of business groups and the policies needed to improve the efficiency of economy-wide capital allocation.

References

Almeida, Heitor, and Murillo Campello, 2007, Financial constraints, asset tangibility, and corporate investment, *The Review of Financial Studies* 20, 1429–1460.

Almeida, Heitor, Chang-Soo Kim, and Hwanki Brian Kim, 2015, Internal capital markets in business groups: Evidence from the asian financial crisis, *The Journal of Finance* 70, 2539–2586.

Almeida, Heitor, and Daniel Wolfenzon, 2006a, Should business groups be dismantled? the equilibrium costs of efficient internal capital markets, *Journal of Financial Economics* 79, 99–144.

Almeida, Heitor, and Daniel Wolfenzon, 2006b, A theory of pyramidal ownership and family business groups, *The Journal of Finance* 61, 2637–2680.

Belenzon, Sharon, Honggi Lee, and Andrea Patacconi, 2018, Towards a legal theory of the firm: The effects of enterprise liability on asset partitioning, decentralization and corporate group growth, *Working Paper* 1–76.

Berger, Philip G, and Eli Ofek, 1995, Diversification's effect on firm value, *Journal of Financial Economics* 37, 39–65.

Berle, A., and G. Means, 1932, *The Modern Corporation and Private Property* (Macmillan).

Bernanke, Ben, and Mark Gertler, 1989, Agency costs, net worth, and business fluctuations, *The American Economic Review* 79, 14–31.

Bertrand, Marianne, Paras Mehta, and Sendhil Mullainathan, 2002, Ferreting out tunneling: an application to Indian business groups, *The Quarterly Journal of Economics* 117, 121–148.

Billett, Matthew T., and David C. Mauer, 2003, Cross-subsidies, external financing constraints, and the contribution of the internal capital market to firm value, *The Review of Financial Studies* 16, 1167–1201.

Buchuk, David, Borja Larrain, Francisco Muñoz, and Francisco Urzúa I., 2014, The internal capital markets of business groups: Evidence from intra-group loans, *Journal of Financial Economics* 112, 190–212.

Campello, Murillo, 2002, Internal capital markets in financial conglomerates: Evidence from small bank responses to monetary policy, *The Journal of Finance* 57, 2773–2805.

Carney, R.W., and T.V. Child, 2012, Changes to the ownership and control of east asian corporations between 1996 and 2008: The primacy of politics, *Journal of Financial Economics* 107, 494–513.

Cestone, Giacinta, and Chiara Fumagalli, 2005, The strategic impact of resource flexibility in business groups, *The RAND Journal of Economics* 36, 193–214.

Chong, A., and F. López-de Silanes, 2007, *Investor protection and corporate governance: firm-level evidence across Latin America* (World Bank).

Claessens, S., S. Djankov, J. Fan, and L. Lang, 2002, Disentangling the incentive and entrenchment effects of large shareholdings, *Journal of Finance* 52, 2741–2771.

DeMarzo, Peter M., and Michael J. Fishman, 2007, Agency and optimal investment dynamics, *The Review of Financial Studies* 20, 151–188.

Faccio, Mara, and Larry H.P. Lang, 2002, The ultimate ownership of western european corporations, *Journal of Financial Economics* 65, 365–395.

Fazzari, Steven M., R. Glenn Hubbard, and Bruce C. Petersen, 1988, Financing constraints and corporate investment, *Brookings Papers on Economic Activity* 1, 141–195.

Gertler, Mark, and Sim Gilchrist, 1994, Monetary policy, business cycles, and the behavior of small manufacturing firms, *The Quarterly Journal of Economics* 109, 309–340.

Gopalan, Radhakrishnan, Vikram Nanda, and Amit Seru, 2007, Affiliated firms and financial support: evidence from Indian business groups, *Journal of Financial Economics* 86, 759–795.

Gopalan, Radhakrishnan, and Kangzhen Xie, 2011, Conglomerates and industry distress, *The Review of Financial Studies* 24, 3642–3687.

Holmstrom, Bengt, and Jean Tirole, 1997, Financial intermediation, loanable funds, and the real sector, *The Quarterly Journal of Economics* 112, 663–691.

Hoshi, Takeo, Anil Kashyap, and David S. Scharfstein, 1991, Corporate structure, liquidity, and investment: evidence from Japanese industrial groups, *The Quarterly Journal of Economics* 106, 33–60.

Hovakimian, Gayané, 2011, Financial constraints and investment efficiency: Internal capital allocation across the business cycle, *Journal of Financial Intermediation* 20, 264–283.

Johnson, Simon, Rafael La Porta, Florencio Lopez-de Silanes, and Andrei Shleifer, 2000, Tunneling, *The American Economic Review* 90, 22–27.

Kandel, Eugene, Konstantin Kosenko, Randall Morkc, and Yishay Yafeh, 2018, The great pyramids of america: A revised history of u.s. business groups, corporate ownership, and regulation, 1926–1950, *Strategic Management Journal* 40, 781–808.

Kaplan, Steven N., and Luigi Zingales, 1997, Do investment-cash flow sensitivities provide useful measures of financing constraints?, *The Quarterly Journal of Economics* 112, 169–215.

Khanna, Tarun, and Krishna Palepu, 2000, Is group affiliation profitable in emerging markets? An analysis of diversified Indian business groups, *The Journal of Finance* 55, 867–891.

Khanna, Tarun, and Yishay Yafeh, 2007, Business groups in emerging markets: paragons or parasites?, *Journal of Economic Literature* 45, 331–372.

Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy* 105, 211–248.

Kuppuswamy, Venkat, and Belén Villalonga, 2016, Does diversification create value in the presence of external financing constraints? evidence from the 2007–2009 financial crisis, *Management Science* 62, 905–923.

La Porta, Rafael, Florencio Lopez-de Silanes, and Andrei Shleifer, 1999, Corporate ownership around the world, *The Journal of Finance* 54, 471–517.

Lamont, Owen, 1997, Cash flow and investment: evidence from internal capital markets, *The Journal of Finance* 52, 83–109.

Lee, Sangwoo, Kwangwoo Park, and Hyun-Han Shin, 2009, Disappearing internal capital markets: evidence from diversified business groups in korea, *Journal of Banking & Finance* 33, 326–334.

Matvos, Gregor, and Amit Seru, 2014, Resource allocation within firms and financial market dislocation: evidence from diversified conglomerates, *The Review of Financial Studies* 27, 1143–1189.

Morck, Randall, Daniel Wolfenzon, and Bernard Yeung, 2005, Corporate governance, economic entrenchment, and growth, *Journal of Economic Literature* 43, 655–720.

Ozbas, Oguzhan, and David S. Scharfstein, 2010, Evidence on the dark side of internal capital markets, *The Review of Financial Studies* 23, 581–599.

Rajan, Raghuram, Henri Servaes, and Luigi Zingales, 2000, The cost of diversity: the diversification discount and inefficient investment, *The Journal of Finance* 55, 35–80.

Samphantharak, Krislert, 2006, Internal capital markets in business groups, Working Paper 975562, SSRN, Available at SSRN.

Scharfstein, David S., and Jeremy C. Stein, 2000, The dark side of internal capital markets: divisional rent-seeking and inefficient investment, *The Journal of Finance* 55, 2537–2564.

Shin, Hyun-Han, and Young S. Park, 1999, Financing constraints and internal capital markets: evidence from korean 'chaebols', *Journal of Corporate Finance* 5, 169–191.

Shin, Hyun-Han, and René M. Stulz, 1998, Are internal capital markets efficient?, *The Quarterly Journal of Economics* 113, 531–552.

Siegel, Jordan, and Prithwiraj Choudhury, 2012, A reexamination of tunneling and business groups: new data and new methods, *The Review of Financial Studies* 25, 1763–1798.

Stein, Jeremy C., 1997, Internal capital markets and the competition for corporate resources, *The Journal of Finance* 52, 111–133.

Stein, Jeremy C., 2003, Chapter 2 - agency, information and corporate investment, in G. M. Constantinides, M. Harris, and R. Stulz, eds., *Handbook of the Economics of Finance*, volume 1, Part A, 111–165 (Elsevier).

Stiglitz, Joseph E., and Andrew Weis, 1981, Credit rationing in markets with imperfect information, *The American Economic Review* 71, 393–410.

Tirole, Jean, 2006, *The theory of corporate finance* (Princeton University Press).

Williamson, Oliver E., 1975, *Markets and hierarchies: analysis and antitrust implications* (The Free Press).

Figure 1: Timing of the model

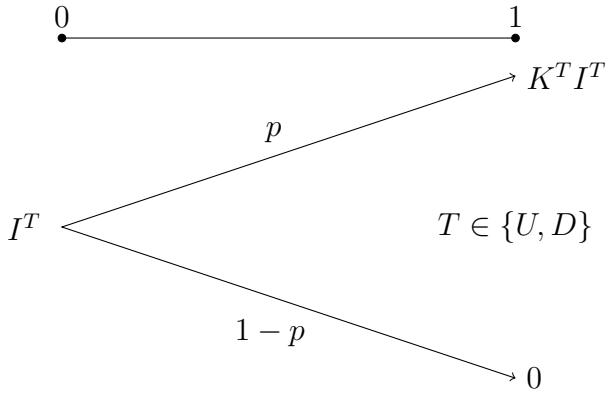


Figure 2: Internal capital market (ICM) outcomes and productivity differences.

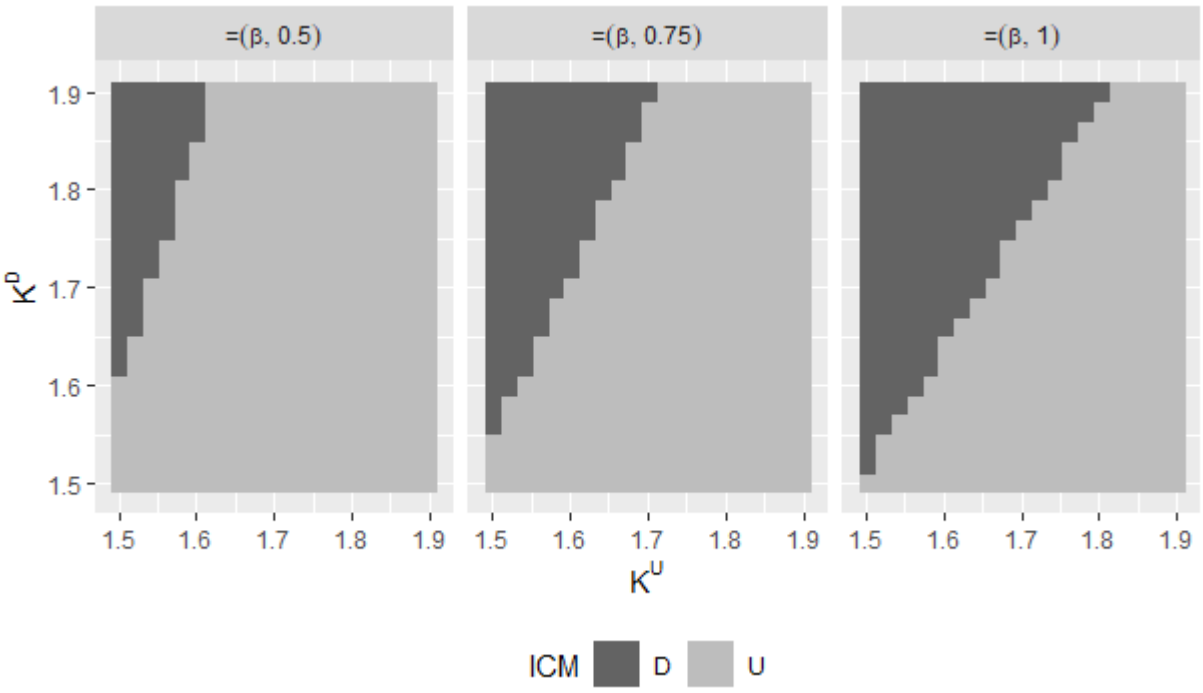
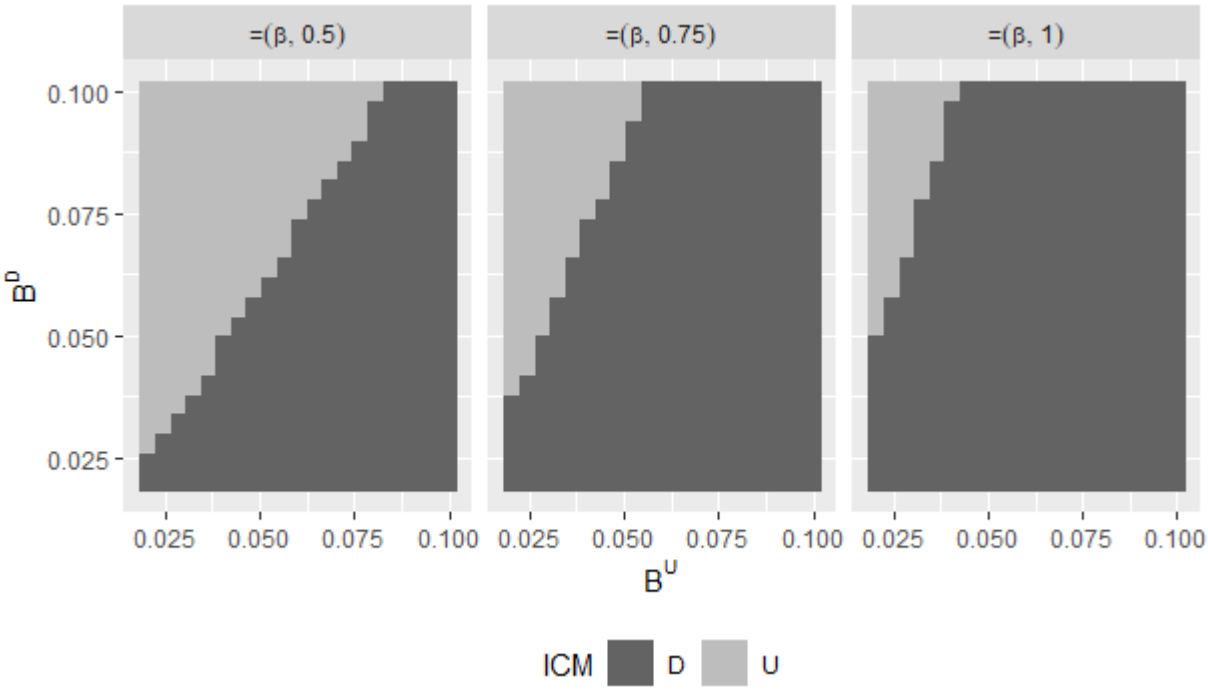


Figure 3: Internal capital market (ICM) outcomes and private benefit differences.



A Appendix

In this Appendix, we develop a more general model than the one presented in the main text. It is worth noting that our key predictions about the resource allocation in business groups do not depend on the assumption of the production technology. Instead, our model is based on the assumption that pledgeable income is not enough to fund new investments and firms must complemented with internal resources. In other words, firms are financially constrained in the sense that external finance and investment depend on the amount of internal resources available to the firms. For business groups, this assumption implies a limit on the amount of the resources that can be transferred across group firms. We also assume that when a group-affiliated firm approaches outside investors it depends solely on its own merits. Thus, it may be optimal for business groups to allocate internal wealth to firms with the greatest capacity to multiply this wealth; in other words, firms with high productivity (high profitability) **and** pledgeable income (external finance capacity).

Specifically, we assume now that the investment cash flow at date 1 in case of success is $f^U(I^U)$ for firm U and $f^D(I^D)$ for firm D. Except for some adjustments in the assumptions (A1) and (A2), everything else in the model setup remain the same. Regarding the production technology, we will assume the following:

$$\begin{aligned} &f^U(0) = 0,\ f^U_I(\cdot) > 0,\ f^U_{II}(\cdot) \leq 0,\ \text{and } p_H f^U_I(0) > 1, \\ &f^D(0) = 0,\ f^D_I(\cdot) > 0,\ f^D_{II}(\cdot) \leq 0,\ \text{and } p_H f^D_I(0) > 1. \end{aligned} \tag{A0}$$

In the case of decreasing returns to scale, namely, $f^T_{II}(\cdot) < 0$ for $T \in \{U, D\}$, we define the first-best investments as the ones that satisfy $p_H f^U_I(I^{U,FB}) = 1$ for firm U and $p_H f^D_I(I^{D,FB}) = 1$ for firm D. For $I^U < I^{U,FB}$ and $I^D < I^{D,FB}$, $p_H f^U_I(I^U) > 1$ and $p_H f^D_I(I^D) > 1$ under (A0). With these definitions and changes, assumption (A1) needs to be replaced by:

$$\begin{aligned} p_L f_I^U(I^U) + B^U &< 1, \\ p_L f_I^D(I^D) + B^D \div \beta &< 1. \end{aligned} \tag{A1$'$}$$

In order to have solutions in which firms are financially constrained, assumption (A2) also requires modifications being replaced by:

$$\begin{aligned} p_H \left(f_I^U(0) - \frac{B^U}{\Delta_p} \right) &< 1, \\ p_H \left(f_I^D(0) - \frac{B^D}{\beta \Delta_p} \right) &< 1. \end{aligned} \tag{A2$'$}$$

The investment of firm U (D) will be financed by a fraction α^U (α^D) of its cash flow A^U (A^D), by a fraction $(1-\alpha^D)$ ($(1-\alpha^U)$) of the cash flow of firm D (U), and the remaining by external finance. Under direct loan, the borrowing firm, say U, needs to promise an amount of $(1-\alpha^D)A^D \div p_H$ in the case of success at date 1 in exchange of a loan of $(1-\alpha^D)A^D$ at date 0. The incentive compatibility constraint of the entrepreneur in the case of firm U is:

$$\begin{aligned} &p_H \left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) + (1-p_H)R_F^U \geq \\ &p_L \left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) + (1-p_L)R_F^U + B^U I^U. \end{aligned}$$

Simplifying:

$$\Delta_p \left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) \geq B^U I^U. \tag{IC^U}$$

The entrepreneur's incentive compatibility constraint, in the case of firm D, is:

$$p_H\left(\beta R_S^D+(1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right)+(1-p_H)\beta R_F^D\geq$$
$$p_L\left(\beta R_S^D+(1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right)+(1-p_L)\beta R_F^D+B^DI^D.$$

Simplifying:

$$\Delta_p\left(\beta(R_S^D-R_F^D)+(1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right)\geq B^DI^D. \qquad (IC^D)$$

The investor's rationale constraint, in the case of firm U, is:

$$p_H\left(f^U(I^U)-R_S^U\right)-(1-p_H)R_F^U\geq I^U-\alpha^UA^U-(1-\alpha^D)A^D. \qquad (IR^U)$$

The investor;s rationale constraint, in the case of firm D, is:

$$p_H\left(f^D(I^D)-R_S^D\right)-(1-p_H)R_F^D\geq I^D-\alpha^DA^D-(1-\alpha^U)A^U. \qquad (IR^D)$$

Finally, the entrepreneur wants to maximize its expected total income:

$$p_H\left(R_S^U-(1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right)+(1-p_H)R_F^U+$$
$$p_H\left(\beta R_S^D+(1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right)+(1-p_H)\beta R_F^D.$$

The Lagrangian of the problem:

$$
\begin{aligned}
L = {} & p_H \left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) + (1-p_H)R_F^U \\
& + p_H \left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H} \right) + (1-p_H)\beta R_F^D \\
& - \lambda^U \left[B^U I^U - \Delta_p \left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H} \right) \right] \\
& - \lambda^D \left[B^D I^D - \Delta_p \left(\beta(R_S^D - R_F^D) + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H} \right) \right] \\
& - \theta^U \left[I^U - \alpha^U A^U - (1-\alpha^D)A^D - p_H \left(f^U(I^U) - R_S^U \right) + (1-p_H)R_F^U \right] \\
& - \theta^D \left[I^D - \alpha^D A^D - (1-\alpha^U)A^U - p_H \left(f^D(I^D) - R_S^D \right) + (1-p_H)R_F^D \right] \\
& + \pi_S^U R_S^U + \pi_F^U R_F^U + \pi_S^D R_S^D + \pi_F^D R_F^D + \psi^U I^U + \psi^D I^D \\
& - \delta^U(\alpha - \alpha^U) - \delta^D(\alpha - \alpha^D) - \phi^U(\alpha^U - 1) - \phi^D(\alpha^D - 1).
\end{aligned}
$$

The First-Order Conditions (FOCs):

$$\frac{\partial L}{\partial R_S^U} = p_H + \lambda^U \Delta_p - \theta^U p_H + \pi_S^U = 0, \tag{A.1}$$

$$\frac{\partial L}{\partial R_F^U} = (1-p_H) - \lambda^U \Delta_p - \theta^U (1-p_H) + \pi_F^U = 0, \tag{A.2}$$

$$\frac{\partial L}{\partial R_S^D} = p_H \beta + \lambda^D \Delta_p \beta - \theta^D p_H + \pi_S^D = 0, \tag{A.3}$$

$$\frac{\partial L}{\partial R_F^D} = (1-p_H)\beta - \lambda^D \Delta_p \beta - \theta^D (1-p_H) + \pi_F^D = 0, \tag{A.4}$$

$$\frac{\partial L}{\partial I^U} = -\lambda^U B^U - \theta^U \left(1 - p_H f_I^U(I^U) \right) + \psi^U = 0, \tag{A.5}$$

$$\frac{\partial L}{\partial I^D} = -\lambda^D B^D - \theta^D \left(1 - p_H f_I^D(I^D) \right) + \psi^D = 0, \tag{A.6}$$

$$\frac{\partial L}{\partial \alpha^U} = -(1-\beta)A^U - \frac{\lambda^D \Delta_p (1-\beta)A^U}{p_H} + \theta^U A^U - \theta^D A^U + \delta^U - \phi^U = 0, \tag{A.7}$$

$$\frac{\partial L}{\partial \alpha^D} = (1-\beta)A^D + \frac{\lambda^U \Delta_p (1-\beta)A^D}{p_H} - \theta^U A^D + \theta^D A^D + \delta^D - \phi^D = 0. \tag{A.8}$$

Since the lender sector is competitive, firms will earn the entire surplus. Under our assumptions, this means that both firms invest up to their first-best levels, $0 < I^U \leq I^{U,FB}$ and $0 < I^D \leq I^{D,FB}$, and that the investor's rationality constraints are binding at the optimum. Regarding the firms payoffs, in each state (success of failure), there is four possible cases: 1) $R_S^T = R_F^T = 0$, 2) $R_S^T > 0$ and $R_F^T > 0$, 3) $R_S^T = 0$ and $R_F^T > 0$, and 4) $R_S^T > 0$ and $R_F^T = 0$, $T \in \{U, D\}$. The first case is clearly not optimal. If we make the additional assumption that both firms are financial constrained, internal wealth ($A^U + A^D$) and external finance are low enough to not allow firms to invest at the first-best levels[18], we can show that the second and third cases are also ruled out. If $R_S^U > (=) \ 0$ and $R_F^U > 0$, then $\pi_S^U = (\geq) \ 0$ and $\pi_F^U = 0$. Hence, from (A.1) and (A.2):

$$p_H \left(\theta^U - 1\right) = (\geq) \ \lambda^U \Delta_p \geq 0,$$
$$(1 - p_H) \left(\theta^U - 1\right) = -\lambda^U \Delta_p \leq 0.$$

These conditions can only be satisfied if $\lambda^U = 0$, that is, if the incentive compatibility constraint (IC^U) is not binding. As we will show below, this only happens when firm U is financial unconstrained. In the more interesting scenario of financial constraints (and even more realistic!), $\lambda^U > 0$ implying that $R_S^U > 0$ and $R_F^U = 0$ (the forth case above). The same applies to firm D. If $R_S^D > (=) \ 0$ and $R_F^D > 0$, then $\pi_S^D = (\geq) \ 0$ and $\pi_F^D = 0$. Hence, from

[18] Under constant returns to scale (main text), firms are always financial constrained.

(A.3) and (A.4):

$$p_H\left(\theta^D-\beta\right)=(\geq)\ \lambda^D\Delta_p\beta\geq 0,$$
$$(1-p_H)\left(\theta^D-\beta\right)=-\lambda^D\Delta_p\beta\leq 0.$$

Again, if firm D is financial constrained then $\lambda^D>0$ implying that $R_S^D>0$ and $R_F^D=0$. That is, if firms are financial constrained they are reward only in the case of success, providing the right incentives for the entrepreneur to behave. From now on we will assume that this is the case. Knowing that $\psi^U=\psi^D=\pi_S^U=\pi_S^D=0$, we can use the first order-conditions (A.1)-(A.6) to solve for θ^U, θ^D, λ^U, λ^D, π_F^U, and π_F^D:

$$\theta^U=\frac{p_H B^U/\Delta_p}{1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta_p}\right]}, \tag{A.9}$$

$$\theta^D=\frac{p_H B^D/\Delta_p}{1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]}, \tag{A.10}$$

$$\lambda^U=\frac{p_H}{\Delta_p}\times\frac{p_H f_I^U(I^U)-1}{1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta_p}\right]}, \tag{A.11}$$

$$\lambda^D=\frac{p_H}{\Delta_p}\times\frac{p_H f_I^D(I^D)-1}{1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]}, \tag{A.12}$$

$$\pi_F^U=\frac{p_H f_I^U(I^U)-1}{1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta_p}\right]}, \tag{A.13}$$

$$\pi_F^D=\frac{\beta\left[p_H f_I^D(I^D)-1\right]}{1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]}. \tag{A.14}$$

The incentive compatibility constraints, (IC^U) and (IC^D), are binding at the optimal solution and determine the firms payoffs in case of success:

$$R_S^U = \frac{B^U I^U}{\Delta_p} + \frac{(1-\beta)(1-\alpha^D)A^D}{p_H}, \tag{A.15}$$

$$R_S^D = \frac{B^D I^D}{\beta \Delta_p} - \frac{(1-\beta)(1-\alpha^U)A^U}{\beta p_H}. \tag{A.16}$$

After replacing R_S^U and R_S^D by the values given in (A.15) and (A.16), respectively, the investor rationality constraints, (IR^U) and (IR^D), determine (implicitly) the firms investment levels:

$$I^U - p_H \left[f^U(I^U) - \frac{B^U I^U}{\Delta_p} \right] = \alpha^U A^U + \beta(1-\alpha^D)A^D, \tag{A.17}$$

$$\beta \left[I^D - p_H \left[f^D(I^D) - \frac{B^D I^D}{\beta \Delta_p} \right] \right] = \beta \alpha^D A^D + (1-\alpha^U)A^U. \tag{A.18}$$

At this point, it is worth to make some comments about the Lagrange multipliers of our problem (see equations (A.9)-(A.14)). As we know, they measure the increase in the entrepreneur's expected total income if we could relax their respective constraints by a unit (in other words, the shadow values of the constraints). Thus, for example, π_F^U measure (approx.) the value to the entrepreneur if we could set the firm U payoff in case of failure, R_F^U, to the value minus one (instead of zero). More interesting to our analysis are the values of the Lagrange multipliers associated with the investor rationality constraints: θ^U and θ^D. It is easy to show that if the entrepreneur wealth $(A^U + \beta A^D)$ is increased by a unit and this amount is allocated in firm U (D) then the entrepreneur expected total income will increase by θ^U $(\theta^D \div \beta)$. The value added by this marginal wealth allocated in firm U or in firm D (henceforth, value added) is, respectively:

$$\theta^U - 1 = \left(p_H f_I^U(I^U) - 1 \right) \times \frac{1}{1 - p_H \left[f_I^U(I^U) - \frac{B^U}{\Delta_p} \right]}, \tag{A.19}$$

$$\frac{\theta^D}{\beta} - 1 = \beta \left(p_H f_I^D(I^D) - 1 \right) \times \frac{1}{\beta \left[1 - p_H \left[f_I^D(I^D) - \frac{B^D}{\beta \Delta_p} \right] \right]}. \tag{A.20}$$

Thus, the value added is the product of two factors: the first is the expected NPV per unit of investment (at the entrepreneur's perspective) and the second is the firm incremental investment per unit of additional entrepreneur wealth allocated to it (the equity multiplier in the language of Tirole (2006)). As the firms are financially constrained, their investments are restricted to the amounts of internal and external wealth they can attract. So, this last factor depends positively on the pledgeable income of the firm. The amount the firm can raise in the external capital market per unit of entrepreneur wealth allocated to it. With this in mind, the value added is the result of the interaction between productivity (expected NPV) and pledgeable income. As we will see below, the resources in the internal capital market flow to the firm with higher value added.

Now, we need to pin down the optimal values of α^U and α^D. The first-order conditions (A.7) and (A.8) show us how internal transfers impact the entrepreneur expected total income. To see this more clearly, we rewrite these first-order conditions as follows:

$$A^U \times \left[\theta^U - \frac{\theta^D}{\beta} \right] = \phi^U - \delta^U, \tag{A.21}$$

$$\beta A^D \times \left[\frac{\theta^D}{\beta} - \theta^U \right] = \phi^D - \delta^D. \tag{A.22}$$

The left sides of these equations measure the increase in the entrepreneur expected total income if we augment α^U and α^D, respectively, by one unit. Suppose that with no internal transfers ($\alpha^U = \alpha^D = 1$) the value added is higher in firm U than in firm D, that is, $\theta^U > \theta^D \div \beta$. Then equations (A.21) and (A.22) tell us that is worth (in the entrepreneur eyes) transferring some wealth from firm D to firm U, that is, to decrease α^D. As we can see from (A.17) and (A.18), as α^D decreases firm U investment increases, firm D investment decreases, and so the gap between θ^U and $\theta^D \div \beta$ shrinks (see equations (A.9) and (A.10)).

The entrepreneur will continue to transfer internal resources from firm D to firm U until this gap vanishes or until the limit to internal transfers is reached (whichever happens first). In this last case, $\delta^U = \phi^D = 0$ and equations (A.21) and (A.22) determine the values of ϕ^U and δ^D, respectively. Now, suppose that with no internal transfers $\theta^U < \theta^D \div \beta$. Then the direction of resources will be reversed, namely, from firm U to firm D. If the limit to internal transfers is reached before the gap between θ^U and $\theta^D \div \beta$ vanishes, then $\phi^U = \delta^D = 0$ and equations (A.21) and (A.22) determine the values of δ^U and ϕ^D, respectively. Finally, suppose that with no internal transfers $\theta^U = \theta^D \div \beta$. Then the entrepreneur expected total income cannot be increased by internal transfers and the simplest contract is the one with $\alpha^U = \alpha^D = 1$. It is easy to see that in this case $\phi^U = \delta^U = \phi^D = \delta^D = 0$. Therefore, as in the main text, productivity and pledgeable income jointly determine the allocation of resources in the internal capital markets. All of our empirical implications remain valid in this more general context.

Note that we set the upper bond of internal transfers, α, to be greater than zero. This assumption prevents all wealth from one firm from being transferred to the other in the internal capital market and so assures that both firms invest. If we do not impose such a limit, resources will be transferred from one group firm to the other until the gap between their values added vanishes or until there is no more internal wealth to transfer ($\alpha^T = 0$), whichever comes first. In this second case, the "donor" firm does not invest, since it has no wealth to raise resources in the external capital market. However, even in this case, all our predictions remain valid, except those related to the investment-cash flow sensitivities of the "donor" firm.

To better compare the expression used here and the one used in the main text to determine the direction of resources in the internal capital market, it is interesting to note that the expression in brackets on the left side of (A.21) has the same sign as the following expression:

$$B^D \left[p_H f_I^U(I^U) - 1\right] - B^U \beta \left[p_H f_I^D(I^D) - 1\right].$$

In the case of linear technology (constant returns to scale), the expression above is exactly the same as the one used in the main text. Lastly, we check the second-order conditions for a local maximum. If the limit of internal transfers is reached at the optimal solution (no matter the direction of resources), only one condition must be satisfied: the determinant of the (respective) bordered Hessian matrix is positive, that is:

$$\left(\Delta_p\left(1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta}\right]\right)\right)^2\times\left(\beta\Delta_p\left(1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]\right)\right)^2>0.$$

One can see that this condition is satisfied. If the limit of internal transfers is not reached at the optimal solution, then two conditions must be satisfied: the determinant of the (respective) bordered Hessian matrix is positive and the second last leading principal minor is negative. The first condition is not satisfied since this determinant is null. As we can see in the following expression, the second condition is met.

$$p_H(\Delta_p^2A^U)^2\left[\theta^Uf_{II}^U(I^U)\left(\beta\left(1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]\right)\right)^2+\theta^Df_{II}^D(I^D)\left(1-p_H\left[f_I^U(I^U)-\frac{BU}{\Delta_p}\right]\right)^2\right]<0.$$

Hence, there is not a single optimal solution in the sense that once the value added of both firms are equated, changes to α^U and α^D that do not alter firms investment, and neither therefore, the entrepreneur expected total income, are also optimal solutions. We opt for the most parsimonious solution, the one in which the flow of resources is unidirectional.